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A GRAPHICAL SOLUTION OF THE CONVENTIONAL SKILL SCORE INVOLVING TWO CATEGORIES

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ABSTRACT

A chart for the graphical solution of the skill score is developed on the basis that the number of cases is held constant. A procedure is suggested by means of which the chart presented may be used for any number of cases. The overall range of the skill score is shown to be restricted by limiting values of the number of correct and expected correct forecasts. Other characteristic features of the score are pointed out.

1. INTRODUCTION

The conventional skill score is a commonly accepted index for expressing the skill required to attain the accuracy of a given series of forecasts [1, 2]. This score S may be defined as follows:

$$S=\frac{C-E_c}{T-E_c} \tag{1}$$

where T is the total number of cases, C is the number of correct forecasts, and E_c is the number of forecasts expected to be correct on a chance basis, although other bases for the calculation of E_c are occasionally used. The skill score thus defined expresses as a decimal fraction the percentage of forecasts which are correct after those forecasts have been eliminated from consideration which would have been correct on a chance basis.

The evaluation of the score is usually obtained through the use of a contingency table of the symbolic form of table 1, where for convenience here and in the remainder of the paper, the forecasts are considered as being made up of rain and no-rain forecasts. In table 1 and the equations to follow, F_r and F_{nr} are the frequencies of rain and no-rain forecasts, O_r and O_{nr} the observed rain and no-rain days, n_1 and n_2 the correct forecasts, n_3 and n_4 the incorrect forecasts, and T the total number of forecasts. The number of correct forecasts, C, and the number of expected correct forecasts, E_c, are obtained from this table, where

$$C=n_1+n_2 \tag{2}$$

and

$$E_c=\frac{F_rO_r}{T}+\frac{F_{nr}O_{nr}}{T} \tag{3}$$

Although expressed in a simple form, the skill score has many interrelationships among the variables involved. The solution of the score in a graphical form will not only furnish a means for the rapid calculation of the score but will serve to broaden our understanding of it.

TABLE 1.—*Symbolic form of contingency table involving two categories. The skill score is usually evaluated through use of this form of table. See text for definition of symbols.*

		FORECAST		
		Rain	No rain	Total
OBSERVED	Rain	n_1	n_4	O_r
	No rain	n_3	n_2	O_{nr}
	Total	F_r	F_{nr}	T

2. BASIS FOR GRAPHICAL SOLUTION

A contingency table of the type presented (table 1) is completely determined given T, F_r or F_{nr}, O_r or O_{nr}, and any one of the values n_1, n_2, n_3, or n_4. Thus, the table involves four independent variables. The number of variables may be reduced to three by holding the total number of cases constant. For purposes of presentation, T will be considered constant, and F_{nr}, O_{nr}, and n_1 will be taken as the independent variables.

Since $F_r=T-F_{nr}$, and $O_r=T-O_{nr}$, equation (3) may be written

$$E_c=\frac{2F_{nr}O_{nr}}{T}-(F_{nr}+O_{nr})+T \qquad (4)$$

For a given value of O_{nr} there is a linear relationship between E_c and F_{nr}. The solutions for E_c, for $T=100$, are entered on figure 1 as a family of straight lines for values of O_{nr} ranging from 0 to 100 at intervals of 10.

Expressing (1) in terms of C, we may write

$$C=E_c(1-S)+ST \qquad (5)$$

For given values of S, solutions of this expression are a family of straight lines with slopes $(1-S)$ and with ordinate intercepts ST. Solutions to (5) for values of S ranging from 1.00 to -1.00, for $T=100$, have been entered on figure 1. For the given value of T, the value of S may now be determined graphically from the figure for any possible combination of F_{nr}, O_{nr}, and C.

3. RANGE OF SKILL SCORE RESTRICTED BY DATA

An examination of table 1 indicates that for given values of F_{nr}, O_{nr}, and T, the values of the variables n_1 and n_2 are restricted to certain ranges which in turn restrict the range in the value of the skill score.[1] In other words, for a given C the range in S is limited by the range in E_c. From relationships obtained from the contingency table we may write

$$C=(F_{nr}+O_{nr})-T+2n_1, \quad (F_{nr}+O_{nr}+2n_1\geqq T) \qquad (6)$$

$$C=T-(F_{nr}+O_{nr})+2n_2, \quad (T+2n_2\geqq F_{nr}+O_{nr}) \qquad (7)$$

where the associated restrictions are due to the fact that C cannot take on negative values. For given values of F_{nr} and O_{nr}, the value of C is determined by n_1 or n_2. Therefore, C takes on its minimum value when $n_1=0$ in (6) or $n_2=0$ in (7). Thus

$$C_{min}=(F_{nr}+O_{nr})-T, \quad (F_{nr}+O_{nr}\geqq T) \qquad (8)$$

$$C_{min}=T-(F_{nr}+O_{rn}), \quad (F_{nr}+O_{nr}\leq T) \qquad (9)$$

Solving equation (8) for O_{nr} (equation (9) furnishes the same final result) and substituting into (4) gives

$$E_c=-\frac{2F_{nr}^2}{T}+\frac{2F_{nr}(C_{min}+T)}{T}-C_{min} \qquad (10)$$

For a given value of C_{min}, i. e., C_{min} held constant, we may obtain the corresponding limiting value of E_c by differentiating (10) with respect to F_{nr} and equating to zero.

$$\frac{dE_c}{dF_{nr}}=-\frac{4F_{nr}}{T}+\frac{2}{T}(C_{min}+T)=0 \qquad (11)$$

for which E_c takes on a maximum value.

From (11), when E_c is a maximum,

$$F_{nr}=\frac{C_{min}+T}{2} \qquad (12)$$

Combination of (12) with (10) gives

$$E_{c_{max}}=\frac{C_{min}^2}{2T}+\frac{T}{2} \qquad (13)$$

which furnishes the relationship between a given value of C_{min} and the corresponding value of $E_{c_{max}}$. However, for every value of C_{min} ranging from 0 to 100 there is a corresponding value of $E_{c_{max}}$ and equation (13) is a completely general expression giving the relationship between E_c and C when E_c is a maximum for a minimum value of C.

In a similar manner, the relationship between the minimum value of E_c for a maximum value of C can be determined to be

$$E_{c_{min}}=-\frac{C_{max}^2}{2T}+C_{max} \qquad (14)$$

The limiting curves represented by (13) and (14) have been entered on figure 1 from which we see that no solutions are possible in the lower right hand side or in the upper left hand side of the chart.

4. SOME CHARACTERISTIC FEATURES OF THE SKILL SCORE

Several interesting features of the skill score are brought out by figure 1. It is seen that positive and negative values of the score are not symmetrical about any value. As a result there is only one value of E_c for which a score of -1.00 is possible, namely, for $E_c=50$ (or 50 percent of the total number of cases), and it is only for this value of E_c that the score can vary over its full range from -1.00 to 1.00. As E_c increases or decreases from the mid value, the range over which S may vary is rapidly diminished, dropping to negative and small positive values for smaller values of E_c and rising to positive and small negative values for larger values of E_c. As E_c increases, it is seen that a single forecast (or a given percentage of

[1] At a conference of fire-weather forecasters at Portland, Oreg., in February 1954, R. T. Hanna presented empirically derived curves of the upper and lower limits of the skill score versus the percent correct forecasts.

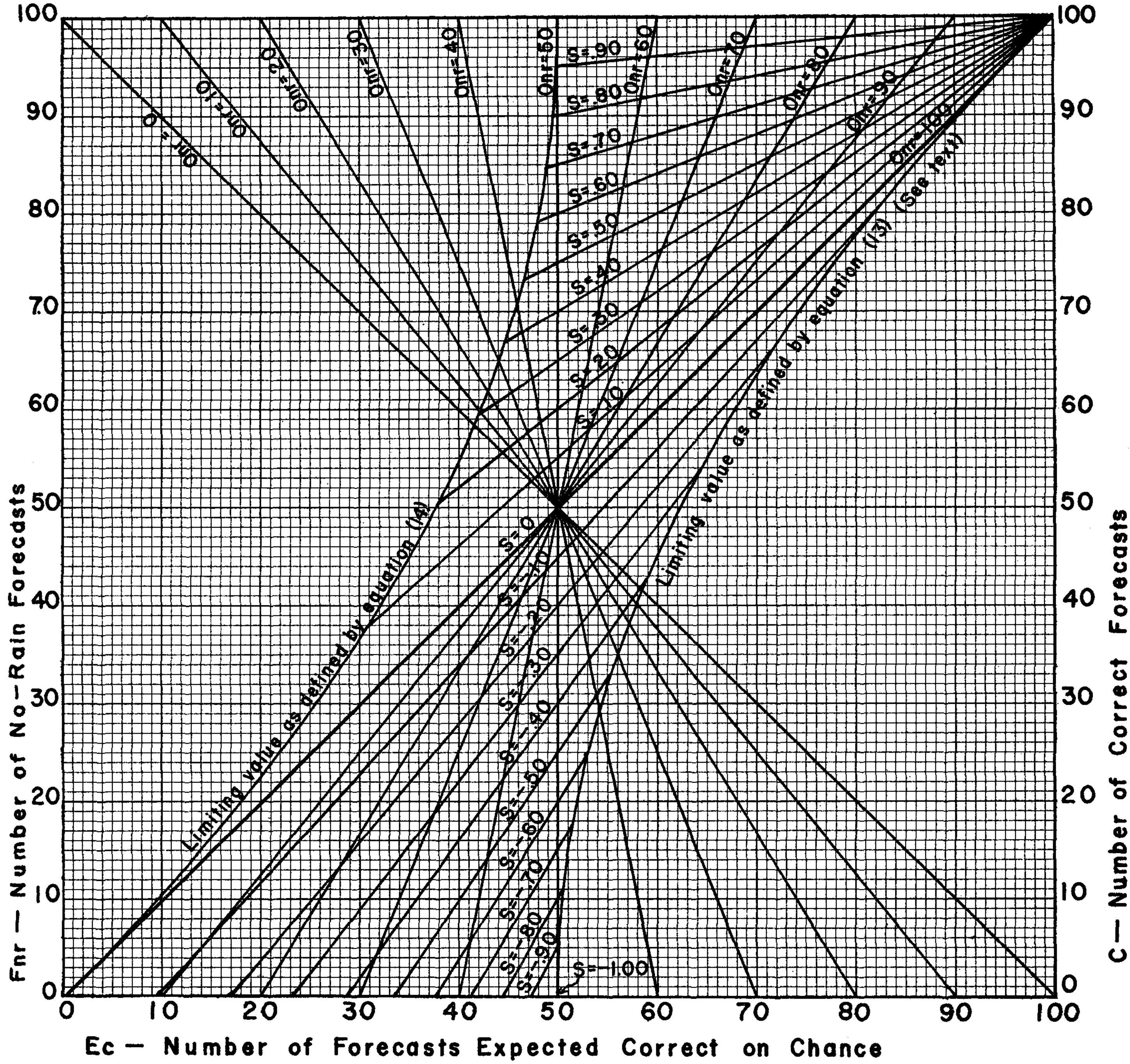

FIGURE 1.—Graphical solution of the skill score for 100 forecasts. To use, enter chart with value of F_{nr}, move over to value of O_{nr} (number of observed no-rain days); from this point which gives the value for E_c, move upward or downward to C, and read the value of the skill score from lines labeled S. The chart may be used for any number of cases by expressing F_{nr}, O_{nr}, and C as percent of total cases.

the forecasts) takes on more and more significance in determining the magnitude of the skill score. In the region of the chart where $E_c=90$ percent of the cases, an increase in the number of correct forecasts by 3 percent (about one day in thirty) will increase the score by 30 points in contrast to an increase of only 6 points at $E_c=50$ percent. In order for an individual forecast to have the same effect in determining the magnitude of the skill score for these two values of E_c, five times as many forecasts are required for the larger value of E_c. Thus, for large percentage values of E_c, the number of forecasts needs to be increased in order to establish the amount of skill with the same degree of certainty.

5. EXAMPLES OF GRAPHICAL SOLUTION

Figure 1 may be used to calculate the skill score given F_{nr}, O_{nr}, and C for a series of 100 forecasts, for example

TABLE 2.—*Contingency table of series of 100 forecasts used as example of calculation of skill score with aid of figure 1*

		FORECAST		
		Rain	No rain	Total
OBSERVED	Rain	20	17	37
	No rain	21	42	63
	Total	41	59	100

TABLE 3.—*Contingency table of series of 46 forecasts used as example of calculation of skill score with aid of figure 1*

		FORECAST		
		Rain	No rain	Total
OBSERVED	Rain	12	6	18
	No rain	8	20	28
	Total	20	26	46

the series given in contingency form in table 2. Entering figure 1 for $F_{nr}=59$ and moving over to the value of $O_{nr}=63$ gives $E_c=52.5$. Following this value up the chart to $C=62$ gives a skill score of 0.20.

Although figure 1 cannot be used directly to obtain the skill score for a number of forecasts different from 100, it can be used if the values of F_{nr}, O_{nr}, and C are expressed as percent of total cases. Thus, for the series given in contingency form in table 3, $C=(32/46)100=69.6$ percent (using slide rule), $F_{nr}=56.5$ percent, and $O_{nr}=60.8$ percent. Using these values to enter figure 1 gives a value of $S=0.375$. The graphical solution of the skill score may be most easily accomplished using a chart designed for the required number of forecasts. Charts similar to figure 1 may be readily prepared for any number of cases.

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